Exotic meson spectroscopy from the clover action at $\beta = 5.85$ and 6.15*

C. Bernard,[a] T. DeGrand,[b] C. DeTar,[c] Steven Gottlieb,[d] U. M. Heller,[e] J. Hetrick,[f] C. McNeile[c], R. Sugar,[g] and D. Toussaint[h]

[a]Department of Physics, Washington University, St. Louis, MO 63130, USA

[b]Department of Physics, University of Colorado, Boulder, CO 80309, USA

[c]Physics Department, University of Utah, Salt Lake City, UT 84112, USA

[d]Department of Physics, Indiana University, Bloomington, IN 47405, USA

[e]SCRI, The Florida State University, Tallahassee, FL 32306-4130, USA

[f]Department of physics, University of the Pacific, Stockton, CA 95211-0197, USA

[g]Department of Physics, University of California, Santa Barbara, CA 93106, USA

[h]Department of Physics, University of Arizona, Tucson, AZ 85721, USA

We repeat our original simulations of the hybrid meson spectrum using the clover action, as a check on lattice artifacts. Our results for the 1^{-+} masses do not substantially change. We present preliminary results for the wave function of the 1^{-+} state in Coulomb gauge.

1. INTRODUCTION

There are currently a number of experimental candidates for light 1^{-+} hybrid mesons [1]. The review of the experimental results by Page [2], suggests that a 1^{-+} state may exist with a mass around 1.6 GeV. The predictions of lattice QCD, by both MILC [3] and UKQCD [4] predict the lightest 1^{-+} state to be around 2 GeV (with large errors). The inclusion of dynamical fermions [5] has not produced results substantially different from those of quenched calculations. To start to check for systematic errors, we have repeated our original simulations [3] of the hybrid meson spectrum that used Wilson fermions, with improved clover quarks.

We have calculated the hybrid meson spectrum at two different β values: 5.85, with a lattice volume of $20^3 \times 48$, and 6.15 with a lattice volume of $32^3 \times 64$. At $\beta = 5.85$ we used a clover coefficient obtained from tadpole improved perturbation theory, using the plaquette value of u_0. At $\beta = 6.15$ we used the non-perturbative value of c_{sw} calculated by the ALPHA collaboration [6].

*Presented by C. McNeile.

2. LIGHT QUARK SPECTROSCOPY

In Table 1, we report our preliminary results for the mass of the 1^{-+} hybrid in the chiral limit. At this stage in our analysis, we have not attempted to estimate the systematic errors in our results from the clover action. To set the scale, we used the value of r_0 from the interpolating formulae published in [7] (this changes the number for the Wilson data slightly from our previously quoted number [3]). Our preliminary results for the light hybrids mesons show that the mass splitting between the 1^{-+} and 0^{+-} state is considerably reduced, relative to our previous results [3] from the simulations using the Wilson action.

3. CHARMONIUM SPECTROSCOPY

We used a more "traditional" [8] approach to analysing our hybrid data at the charm mass, than we originally used in [3]. In Table 2, we

Action	$\kappa_{critical}$
Wilson	$1980(100)_{stat} \pm sys$
clover	$2110(100)_{stat} \pm sys$

Table 1
Mass results for the 1^{-+} state (in MeV) at kappa critical, for the Wilson and clover actions, at $\beta = 6.15$.

Quantity	Wilson	clover
a^{-1}_{P-S}	2650^{+110}_{-90}	2900^{+90}_{-90}
$M_{J/\psi} - M_{\eta_c}$	$27^{+1}_{-1} \pm sys$	$71^{+2}_{-2} \pm sys$
$M_{1^{-+}} - M_S$	$1340^{+60}_{-150} \pm sys$	$1220^{+110}_{-190} \pm sys$
$M_{0^{+-}} - M_S$	$1490^{+110}_{-100} \pm sys$	$1490^{+80}_{-110} \pm sys$

Table 2
Mass splitting results (in MeV) for charmonium for the Wilson and clover actions at $\beta = 6.15$.

present our results for the mass splittings between the hybrids and the spin averaged S-wave mass ($M_S = (M_{\eta_c} + 3M_{J/\psi})/4$), at our kappa value that corresponds to the charm mass. Mass splittings should be less sensitive to lattice artifacts [9,10], than the absolute masses we originally quoted in [3]. Also we used the $P-S$ mass splitting to obtain the lattice spacing. We used the bootstrap method with 100 bootstrap samples to estimate the errors.

In Table 2 we see that the clover action gets the $M_{J/\psi} - M_{\eta_c}$ mass splitting closer to the experimental value of 117 MeV, than the Wilson action. Using potential model ideas El-Khadra [9] estimates the value of this splitting in the quenched approximation to be ~ 70 MeV.

The importance of the heavy-heavy hybrid mesons, and other approaches to studying them, is discussed by Kuti [11].

4. WAVE FUNCTIONS

To study the internal distribution of quarks and glue inside a hybrid meson, we measured

$$\sum_{\underline{x}} \overline{q}(\underline{x}, t) F(\underline{x} + \underline{r}_A + \underline{r}, t) \Gamma q(\underline{x} + \underline{r}, t) \qquad (1)$$

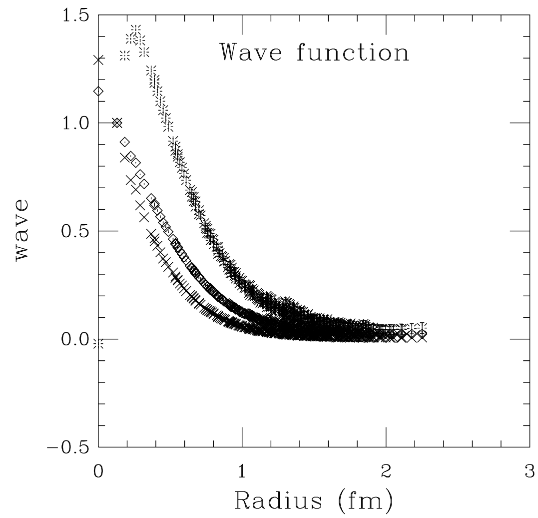

Figure 1. Wave function of the pion (crosses), rho (diamonds) and 1^{-+} state (bursts), at $\beta = 5.85$.

where F is the field strength tensor and we have suppressed the contraction of F with the Γ gamma matrix. We fix to Coulomb gauge, measure a correlator with Eq. 1 at the sink with our standard hybrid meson source [3] at time slice $t = 0$. The operator in Eq. 1 is similar to the one used to measure the wave function of the proton [12], except that one of the quarks is replaced by the field strength tensor. Although the wave function operator is difficult to interpret in terms of constituent gluons, it makes sense in terms of a Fock space analysis.

For our preliminary results we use $\underline{r}_A = 0$. However, it will be interesting to measure the wave function with non-zero $\underline{r}_A$ in Eq. 1. One approach to studying the heavy hybrids is to solve the Schrödinger equation for the quarks with an excited potential, measured in a lattice simulation [11]. The resulting wave functions do not have any dependence on $\underline{r}_A$. A study of the dependence of the operator in Eq. 1, on $\underline{r}_A$, may provide insight into the validity of the excited potential approach to studying hybrid mesons.

In Fig. 1 we plot the wave functions of the pion, rho and 1^{-+} states in Coulomb gauge, at $\beta = 5.85$ and $\kappa = 0.135$ (corresponding to a vector to pseudoscalar mass ratio of 0.85), with a sample size of 40. The scale on the x axis is set using the chirally extrapolated rho mass.

The wave function in Fig. 1 looks qualitatively similar to those obtained [13] by solving the Schrödinger equation with an excited lattice potential (although we are working with lighter quarks).

5. 4-QUARK HYBRID MIXING

An important issue in the spectroscopy of hybrid mesons is to understand the mixing between a hybrid meson and a 4-quark state with the same quantum numbers. We studied this issue by using the valence approximation. The (naive) valence approximation [14] removes

$$-\kappa(1-\gamma_4)U_4(x)\delta_{x,y-\hat{t}} \quad (2)$$

from the quark action, so that the quarks travel forwards in time only (note that NRQCD [16] quarks only propagate forwards in time as well). In the quenched approximation, it is this term that causes mixing between hybrid and 4-quark states via hairpin diagrams (see Fig. 1 in [3]). Liu and collaborators [15] have developed a valence approximation with an improved non-relativistic limit over the prescription in Eq. 2 suitable for the investigation of the relationship between the quark model and QCD. However, for our purposes, the removal of Eq. 2 from the clover action is adequate to study the effect of the hairpin graph on the hybrid spectrum.

In this preliminary study, Fig. 2 shows that the effective mass plots of the 1^{-+} state, in the valence and quenched approximations, look very similar. This suggests that at the parameters at which we are working, mixing via hairpin diagrams between the 1^{-+} hybrid operator and four quark states is a small effect.

This work is supported by the DOE and the NSF. The computations were carried out at CCS (ORNL), CHPC (Utah), and NPACI (SDSC).

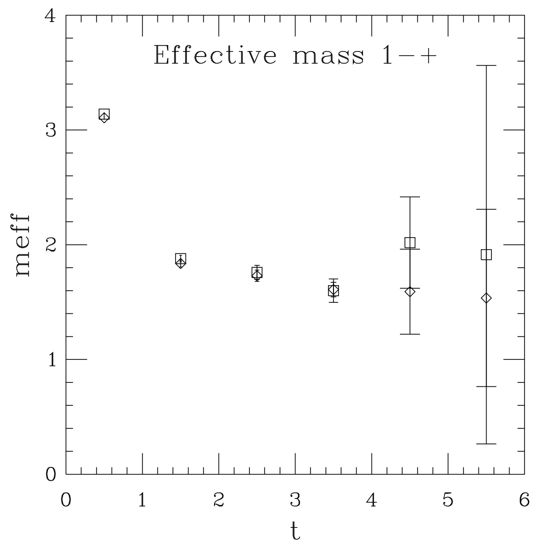

Figure 2. Effective mass plot for the 1^{-+} hybrid in the valence (squares) and quenched (diamonds) approximations, at $\kappa = 0.135$.

REFERENCES

1. F. E. Close, *Nucl. Phys. (Proc. Suppl.)* **63**, 28 (1998).
2. P. R. Page, *Phys. Lett.* **B415**, 205 (1997).
3. C. Bernard *et al.*, *Phys. Rev.* **D56**, 7039 (1997).
4. P. Lacock, C. Michael, P. Boyle, and P. Rowland, *Phys. Lett.* **B401**, 308 (1997). P. Lacock, C. Michael, P. Boyle, and P. Rowland, *Phys. Rev.* **D54**, 6997 (1996).
5. P. Lacock and K. Schilling, these proceedings, hep-lat/9809022.
6. M. Luscher *et al.*, *Nucl. Phys.* **B491**, 323 (1997).
7. M. Guagnelli, R. Sommer, and H. Wittig, hep-lat/9806005, 1998.
8. C. Davies, hep-ph/9710394, 1997.
9. A. X. El-Khadra, *Nucl. Phys. (Proc. Suppl.)* **30**, 449 (1993).
10. A. X. El-Khadra, A. S. Kronfeld, and P. B. Mackenzie, *Phys. Rev.* **D55**, 3933 (1997).
11. J. Kuti, these proceedings.
12. M. W. Hecht and T. A. DeGrand, *Phys. Rev.* **D46**, 2155 (1992).
13. K. J. Juge, J. Kuti, and C. J. Morningstar, *Nucl. Phys. (Proc. Suppl.)* **63**, 326 (1998). S.Collins, C.T.H.Davies, and G.Bali, *Nucl. Phys. (Proc. Suppl.)* **63**, 335 (1998).
14. K.-F. Liu and S.-J. Dong, *Phys. Rev. Lett.* **72**, 1790 (1994).
15. K. F. Liu *et al.*, hep-ph/9806491, 1998.
16. B. Thacker and G. Lepage, *Phys. Rev. D* **43**, 196 (1991).